CALCULATION OF THE HEAT-TRANSFER COEFFICIENT FROM THE TEMPERATURES AT POINTS INSIDE A PLATE

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A method is proposed of determining the heat-transfer coefficient from the instantaneous temperatures at points inside a heated plate.

In the method proposed by the authors of [1] the heat-transfer coefficient is determined experimentally from known temperatures $\varphi_1(\mathrm{Fo})$ and $\varphi_0(\mathrm{Fo})$ at surface points of a plate. If the solution to the equation of heat conduction with boundary conditions of the first kind is written as

$$\Theta(X_1\,\mathrm{Fo}) = \varphi_0(\mathrm{Fo}) + \Phi_0(\mathrm{Fo})\,X + 2\sum_{n=1}^{\infty}\frac{\sin n\pi X}{n\pi}\left[\Phi_n(\mathrm{Fo}) - y_n(\mathrm{Fo})\right], \tag{1}$$

then, according to [1], the following expression can be derived for calculating the heat-transfer coefficient at the surface X = 1:

$$\mathrm{Bi}(\mathrm{Fo}) = \frac{\Phi_0(\mathrm{Fo}) + 2\sum_{n=1}^{\infty}(-1)^n\left[\Phi_n(\mathrm{Fo}) - y_n(\mathrm{Fo})\right]}{\Theta_c(\mathrm{Fo}) - \varphi_1(\mathrm{Fo})}, \tag{2}$$

where

$$\Phi_n(\mathrm{Fo}) = (-1)^n\varphi_1(\mathrm{Fo}) - \varphi_0(\mathrm{Fo}),\ n = 0, 1, \ldots, \tag{3}$$

and functions $y_n(\mathrm{Fo})$ satisfy the ordinary differential equations

$$\frac{1}{(n\pi)^2}\dot{y}_n(\mathrm{Fo}) + y_n(\mathrm{Fo}) = \Phi_n(\mathrm{Fo}),\ n = 1, 2, \ldots \tag{4}$$

Since the time constants $1/(n\pi)^2$ (n = 1, 2, . . .) in (4) decrease rapidly as n increases [2], for approximate calculations of Bi(Fo) one need to consider only a finite number n = s of terms of the infinite summation in (2). To simplify the notation, we omit the argument and obtain the approximate expression for Bi in the following form:

$$\mathrm{Bi}^* = \frac{\Phi_0 + 2\sum_{n=1}^{n=s}(-1)^n(\Phi_n - y_n)}{\Theta_m - \varphi_1}. \tag{5}$$

In several problems involving a determination of the heat-transfer coefficient, however, it is technically difficult to measure temperatures directly on the surface of a plate. In such a case it is possible, at any instant $\mathrm{Fo} = \mathrm{Fo}_j$, to determine approximately the values of the functions in (5) from the temperatures measured at that instant Fo_j at points inside the plate.

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TABLE 1. Error in Calculating the Heat-Transfer Coefficient $\varepsilon = [(Bi^* - Bi)/Bi] \cdot 100\%$

Bi	Fo		
	0,1	0,2	0,3–1,4
2	−11,5	−5,0	−3,5
3	−6,8	−3,7	−2,3
7	−4,9	−2,0	−1,3
20	−2,7	−0,8	−0,4

It follows from expression (1) that the temperature at any point X_i is related to these functions approximately as

$$\Theta(X_i,\ \text{Fo}) = \varphi_0 + \Phi_0 X_i + 2\sum_{n=1}^{n=s} \frac{\sin n\pi X_i}{n\pi}(\Phi_n - y_n). \qquad (6)$$

With the number s fixed, the unknown functions φ_0, Φ_0, and $(\Phi_n - y_n)$ (n = 1, 2, . . . , s) can be determined uniquely on the basis of i = (s + 2) temperature measurements at points inside the plate and by solving the corresponding system of (s + 2) algebraic equations (6). If temperature φ_0 on the surface X = 0 is measured, then the number of equations for the same number of measurements is reduced by one and, consequently, the amount of computations is also less. Temperature φ_1 is determined from the values of functions φ_0 and Φ_0. The minimum number of necessary measurements is three. If temperature φ_0 is measured, then, this yields two algebraic equations and is equivalent to considering only the first term of the finite summation in (2).

A qualitative analysis shows that the error in calculating the heat-transfer coefficient for surface X = 1 from the temperature at surface X = 0 tends toward zero when $X_i \to 1$ (i = 1, . . . , s + 1). This means that all internal measurements must be made close to one another and to the surface X = 1.

In Table 1 are given the results of calculating the Biot number from the temperatures at two internal points $X_1 = 0.90$, $X_2 = 0.95$ and point X = 0 of a plate with a thermally insulated X = 0 surface, these results having been based on the known exact solution with an initial temperature distribution at sink level for Bi = 2, 3, 7, 20, and $\Theta_m = 1$.

We note, in conclusion, that for calculating the heat-transfer coefficient at any instant of time by the proposed method one does not have to know the thermal history of the plate. One must know only the temperatures at points inside the plate at that instant.

NOTATION

Θ temperature of plate;
Θ_m temperature of medium;
x coordinate;
L plate thickness;
a thermal diffusivity;
λ thermal conductivity;
α heat-transfer coefficient;
t time.
$X = x/L$
$Fo = at/L^2$
$Bi(Fo) = \alpha(Fo)L/\lambda$

LITERATURE CITED

1. O. N. Kostelin and L. N. Bronskii, in: Physical Gas Dynamics, Heat Exchange, and High-Temperature Gas Thermodynamics [in Russian], Izd. AN SSSR, Moscow (1962).
2. V. N. Kozlov, Inzh. Fiz. Zh., 16, No. 1 (1969).